Energy-Efficient Transprecision Techniques for Iterative Refinement

Lee, J., Vandierendonck, H., & Nikolopoulos, D. (Accepted/In press). *Energy-Efficient Transprecision Techniques for Iterative Refinement*. Poster session presented at Supercomputing'17 (SC17): International Conference on High Performance Computing, Networking, Storage and Analysis, Denver, United States.

Energy-Efficient Transprecision Techniques for Iterative Refinement

JunKyu Lee, Hans Vandierendonck, Dimitrios S. Nikolopoulos

Centre for Data Science and Scalable Computing, Queen's University of Belfast, UK

Abstract

This work presents transprecision techniques for iterative refinement, which utilize various precision arithmetic dynamically according to numeric properties of the algorithm and computational latencies depending on precisions. The transprecision techniques were plugged into a mixed precision iterative refinement on an Intel Xeon E5-2650 2GHz core with MKL 2017 and XBLAS 1.0. The transprecision techniques brought further 2.0 - 3.4 X speedups and 3.0 - 4.1 X energy reductions to a mixed precision iterative refinement when double precision solution accuracy was required for forward error and a matrix size was ranged from 4K to 32K.

Background

Parallel Computing with *m X* Cores

m X Speedup → m X Power → 1 X Energy

Need some techniques for energy saving? Mixed Precision Method

Mixed precision Iterative Refinement without increasing cores

n X Speedup → 1 X Power → 1/n X Energy

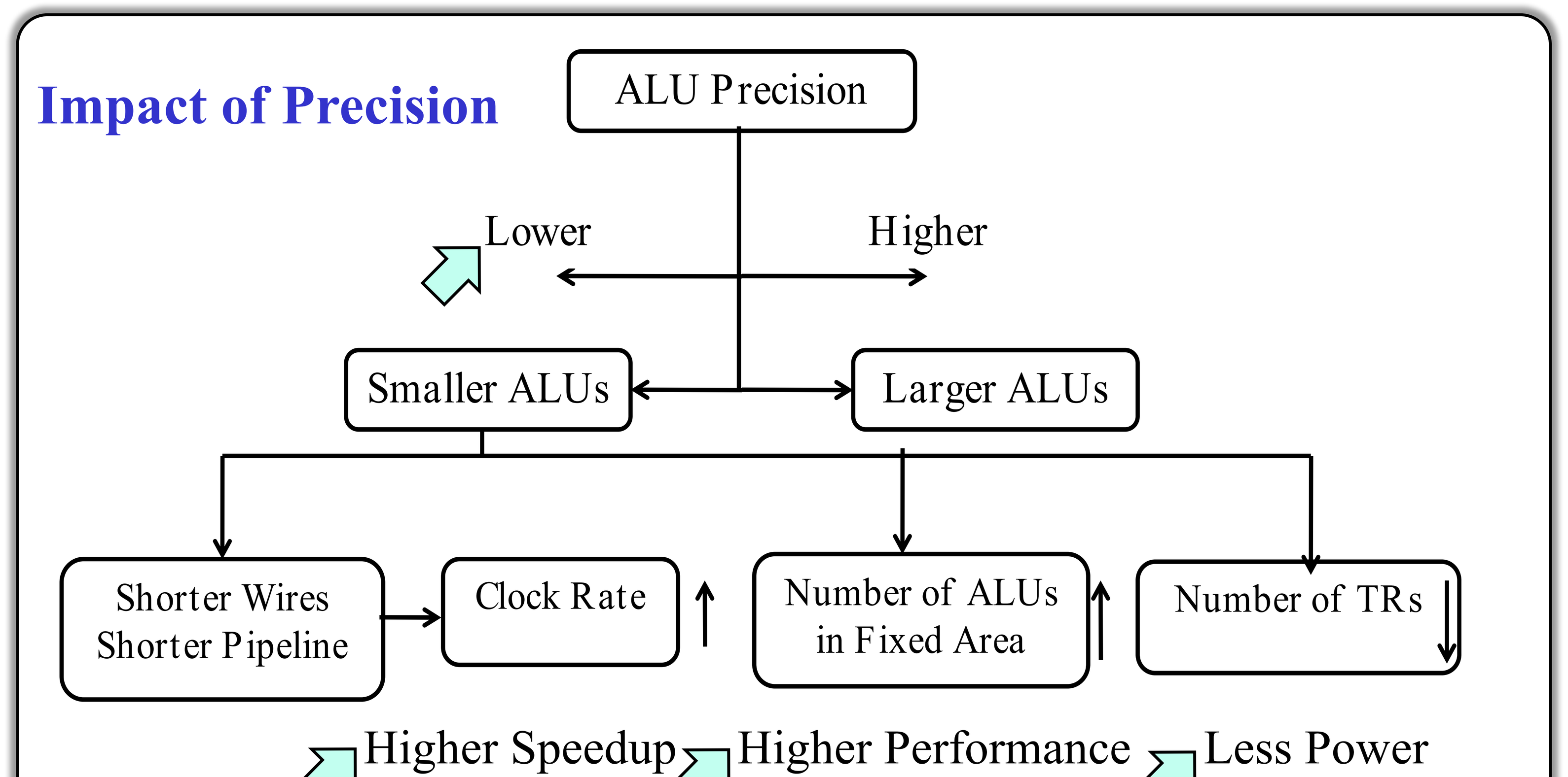
Nice, but further energy saving? Transprecision Techniques!

Transprecision Techniques for Mixed precision Iterative Refinement

(s × n) X Speedup → 1 X Power → 1/(s × n) X Energy

s times future energy saving to Mixed precision Iterative Refinement

Impact of Precision



Transprecision Techniques

Methodology

Algorithm Numeric Properties → Transprecision Techniques ← Latencies depending on Precisions

Transprecision Techniques → Plug them into mixed precision iterative refinement

Mixed-IR : Double precision accuracy for forward error

Approximation

Step 1: LUPP — $O(n^3)$, ϵ_S for Mixed-IR, ϵ_D for Uni-IR

$LU \times \mathbf{x}^{(1)} = P \times \mathbf{b}$ — $O(n^2)$, ϵ_S for Mixed-IR, ϵ_D for Uni-IR

Refinement

Step 2: $\mathbf{r}^{(i)} = A \times \mathbf{x}^{(i)} - \mathbf{b}$ — $O(n^2)$, ϵ_{DD} **(TT 1) ϵ_2^{init} = double**

Step 3: $LU \times \mathbf{z}^{(i)} = P \times \mathbf{r}^{(i)}$ — $O(n^2)$, ϵ_1 **(TT 1) if ($\|z\|_\infty / \|z^{prev}\|_\infty > ½$) ϵ_2=dbl-dbl**

Accuracy Check : $\|\mathbf{z}^{(i)}\| / \|\mathbf{x}^{(i)}\| < \epsilon_D$?

(TT 2) if ($l(\epsilon_2) > p \cdot l(\epsilon_{ref\text{-}z})$ && $\epsilon_2 < \epsilon_3^q$ && ϵ_2=dbl-dbl) refine z using double

Step 4: $\mathbf{x}^{(i+1)} = \mathbf{x}^{(i)} - \mathbf{z}^{(i)}$ — $O(n)$, ϵ_{DD}

Go back to Step 2 **(TT 3) if (NP3) exit(Success);**

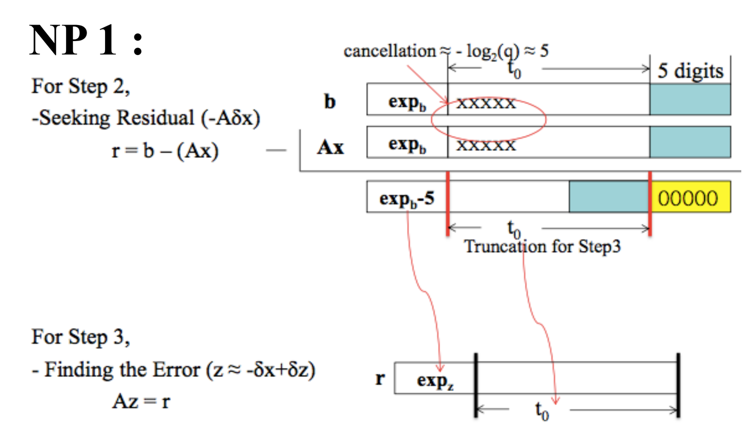
LUPP: LU factorization with Partial Pivoting, ϵ_i : Precision for Step i

ϵ_S : Single Precision, ϵ_D: Double Precision, ϵ_{DD}: Double-Double Precision

Numerical Properties → **Transprecison Techniques Plug-In**

Numerical Properties (NP) and Transprecision Techniques (TT)

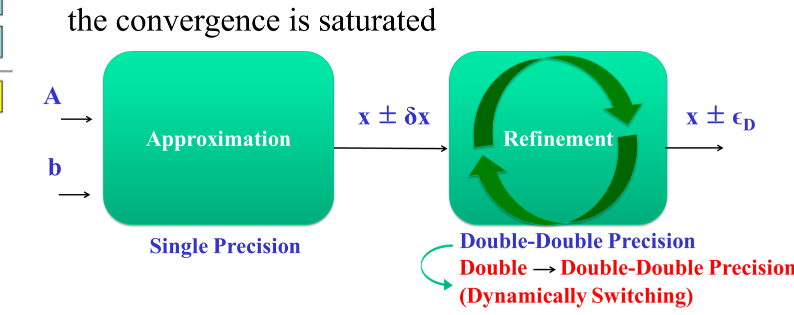
NP 1 :



Residual accuracy is almost kept with the attachment of cancellation bits

TT 1 :

Start double for Step 2 and switch it to dbl-dbl when the convergence is saturated



NP 2 :

Irreducible rounding errors in **r** through Step 3

TT 2 :

Refine **z** if $\epsilon_2 \ll \epsilon_3$ and latency($\epsilon_{ref\text{-}z}$) << latency(ϵ_2)
i.e.) insert an inner loop refinement between Step 3 and 4

NP 3 :

Double precision accuracy guarantied if $\epsilon_1 = \epsilon_S$ and single precision accuracy for **z** is obtained using TT 2

TT 3 :

Skip the accuracy check if the conditions of NP 3 are met

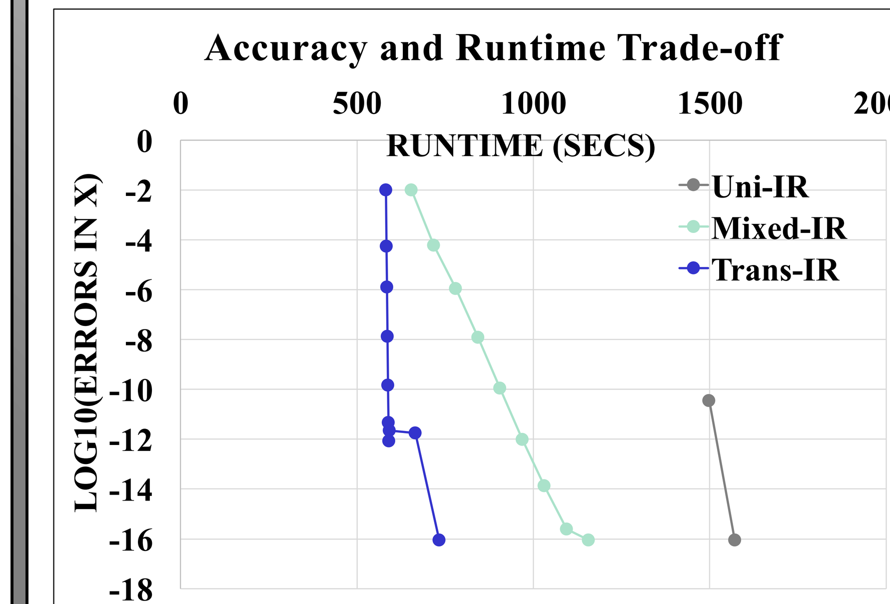
Acknowledgements

This project has received funding by the European Commission Horizon 2020 research and innovation programme under grant agreement No 732631 (OPRECOMP)

Results

Transprecision Techniques on an Intel Xeon E5-2650 2GHz core with MKL 2017 and XBLAS 1.0

Test matrices : Dense uniformly distributed random matrices

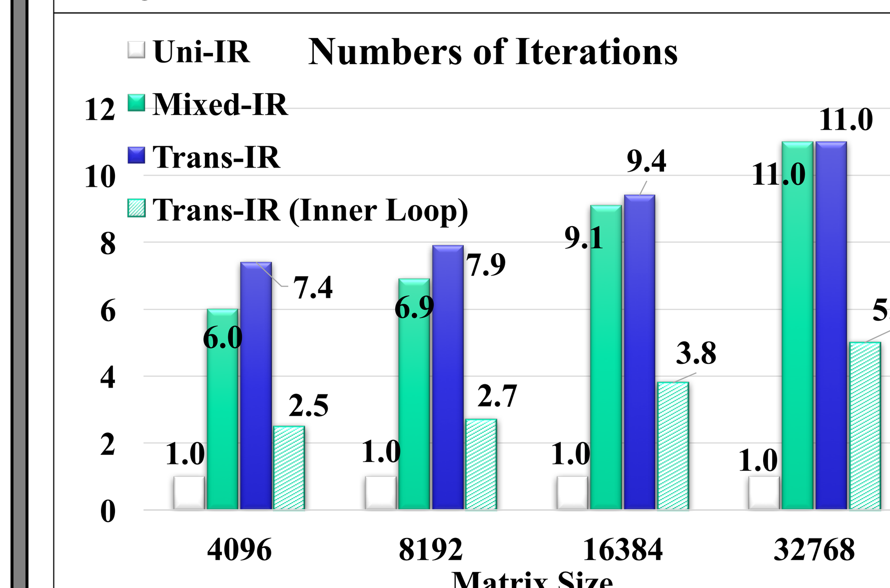


With TTs,

TT 1 achieves an intermediate accuracy (10^{-12}) quicker.

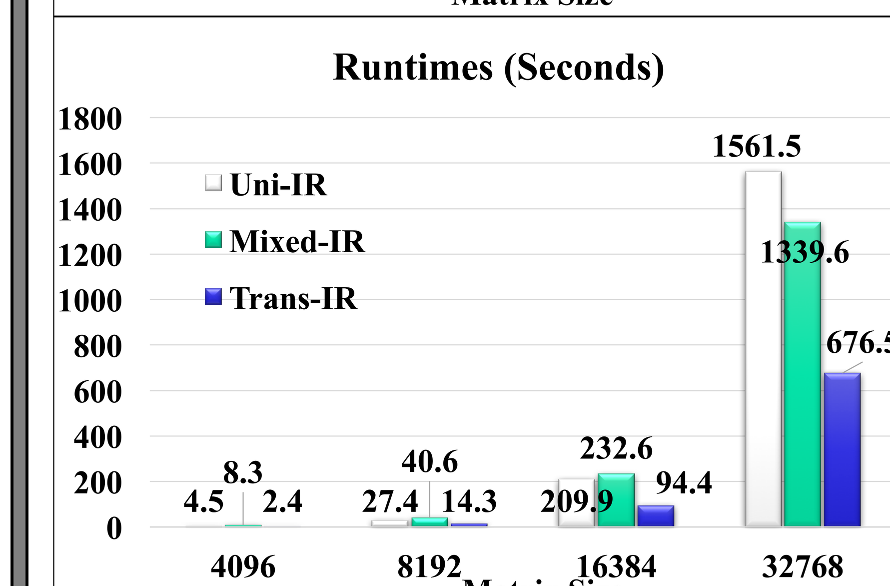
TT 2 enables accuracy to leap from 10^{-12} to 10^{-16}

TT 3 removes the time cost for second dbl-dbl refinement



With TTs,

More iterations in total but, only 1 dbl-dbl iteration using TT 2 and TT 3

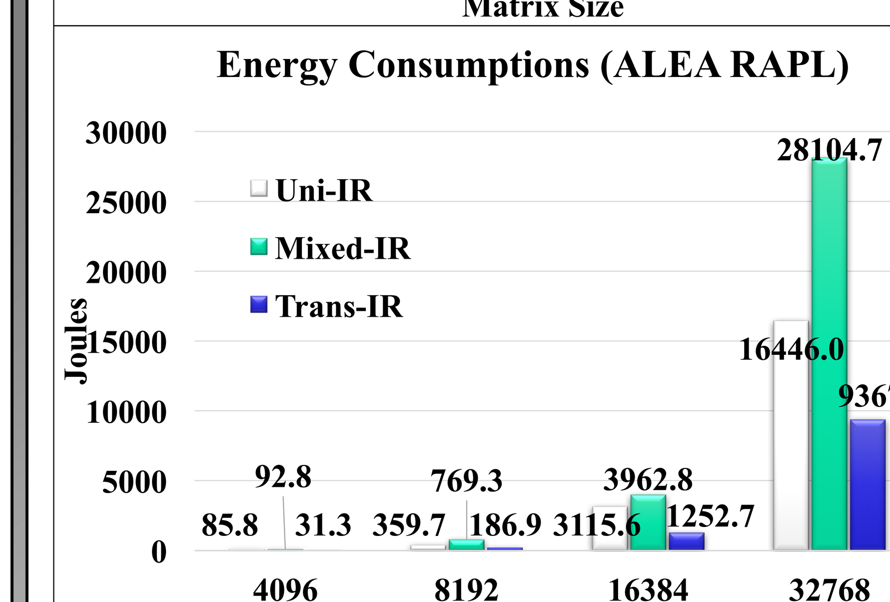


Speedups with TTs

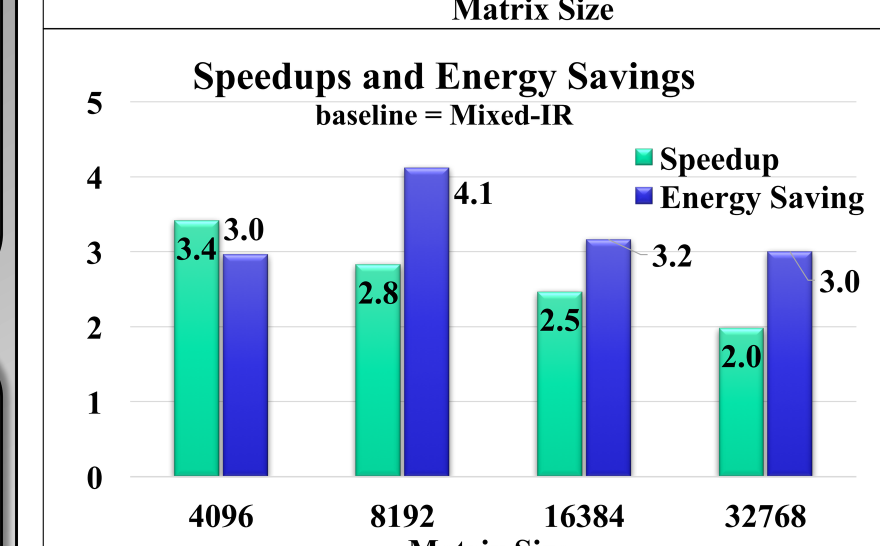
When matrix size N = 32K, Mixed-IR Runtime < Uni-IR

Mixed-IR Runtime ∝ $O(n^{2.5})$
Uni-IR Runtime ∝ $O(n^{2.9})$

Runtime with TTs < Mixed-IR
=> More Energy Saving!



Less Energy with TTs



Transprecision Techniques brought further

2.0 - 3.4 X Speedup
3.0 - 4.1 X Energy Reduction

to Mixed-IR